Problem (Will's Birthday Problem a.k.a. 2016 AIME I #6): In $\triangle ABC$ let I be the center of the inscribed circle, and let the bisector of $\angle ACB$ intersect AB at L. The line through C and L intersects the circumscribed circle of $\triangle ABC$ at the two points C and D. If $LI = 2$ and $LD = 3$, then $IC = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Solution (Andrew Paul with guidance from Bill Li): This problem is difficult but only because it requires an Olympiad lemma.

Lemma (Incenter-Excenter Lemma): Let $\triangle ABC$ have incenter I, A-excenter I_A, and denote by L the midpoint of arc BC. Then, L is the center of a circle through I, I_A, B, and C.

This is shown by angle-chasing. By the Incenter-Excenter Lemma, we have $DA = DB = DI = 2 + 3 = 5$. Next, we observe that since $\angle ALC = \angle DLB$ and $\angle BAC = \angle BDC$ (since $ACBD$ is cyclic), we have $\triangle ACL \sim \triangle DBL$, from which it follows:

$$\frac{AC}{AL} = \frac{DB}{DL} = \frac{5}{3}$$

Now we apply the angle-bisector theorem to $\triangle ACL$ at $\angle CAL$:

$$\frac{AC}{CI} = \frac{AL}{LI}$$

Upon rearrangement, we have:

$$\frac{CI}{LI} = \frac{AC}{AL} \Rightarrow \frac{CI}{2} = \frac{5}{3} \Rightarrow CI = \frac{10}{3}$$

$10 + 3 = \boxed{13}$ and we are done.

Happy Birthday Will :)