Fluid dynamics is an incredibly complicated field. My professor said that it was harder than astrophysics, quantum mechanics, and "all that stuff". Though I don't know much at all, I've figured that it is probably quite difficult. Solving systems of PDEs doesn't sound too easy! With this being said, let us jump right into a problem that we can approach. We let $\vec{F}_D$ denote drag force.

Problem: Assume that $\vec{F}_D \propto \vec{v}$ (this approximation holds only for sufficiently low velocities). Define terminal velocity as:

$$\vec{v}_{\text{max}} = \lim_{t\to\infty} \vec{v}(t)$$

How long will it take for an object in free fall to be within $n\%$ of its terminal velocity if it starts at rest?

Solution: Our first task is to find $\vec{v}(t)$. We can write our net force as:

$$\Sigma\vec{F} = \vec{F}_g + \vec{F}_D$$

We can write the proportionality between $\vec{F}_D$ and $\vec{v}$ as:

$$\vec{F}_D = -b\vec{v}$$

For some $b > 0$ (as drag force acts in the opposite direction of velocity). Now we use our assumed proportionality to rewrite $\vec{F}_D$:

$$\Sigma\vec{F} = \vec{F}_g - b\vec{v}$$

We can also apply Newton's Second Law to rewrite the remaining terms. Considering our desired function is a function of time, we write velocity and acceleration as functions of time:

$$m\vec{a}(t) = m\vec{g} - b\vec{v}(t)$$

Integrating this directly WRT t yields a $\vec{x}(t)$ term that is difficult to reconcile. Instead, we rearrange to obtain:

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \frac{b}{m}\vec{v}(t) = \vec{g}$$

This is a first order linear differential equation. Our integrating factor is $\exp\int \frac{b}{m}\ \mathrm{d}t = e^{\frac{bt}{m}}$. Multiplying our equation through with this yields:

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}e^{\frac{bt}{m}} + \frac{b}{m}\vec{v}(t)e^{\frac{bt}{m}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(e^{\frac{bt}{m}}\vec{v}(t)\right) = \vec{g}e^{\frac{bt}{m}}$$

Now we can integrate to obtain:

$$e^{\frac{bt}{m}}\vec{v}(t) = \vec{g}\int e^{\frac{bt}{m}}\ \mathrm{d}t = \frac{m\vec{g}}{b}e^{\frac{bt}{m}} + C$$

Since the object starts at rest, we have $\vec{v}(0) = 0$, hence:

$$C = -\frac{m\vec{g}}{b}$$

Finally, after some rearrangement, we conclude:

$$\boxed{\vec{v}(t) = \frac{m\vec{g}}{b}\left(1 - e^{-\frac{bt}{m}}\right)}$$

Obviously, $\vec{v}(t)$ is a monotonically increasing function. We find the terminal velocity to be:

$$\frac{m\vec{g}}{b}\lim_{t\to\infty}\left(1-e^{-\frac{bt}{m}}\right)=\frac{m\vec{g}}{b}$$

We wish to find t such that the coefficient of the terminal velocity is $\frac{n}{100}$:

$$1-e^{\frac{-bt}{m}}=\frac{n}{100}$$

Solving for t, we arrive at our solution:

$$\boxed{t=\frac{m}{b}\log\frac{100}{100-n}}$$

As desired. Apparently, the differential equation remained unsolved in my professor's classes for at least three consecutive years! Isn't that a little worrying?