At this point in time, I realize that I am working towards a goal. My goal is to find an equation that when graphed, will give the trajectory of a bouncing projectile given an initial velocity, angle, gravitational acceleration, and coefficient of restitution. But before I do this, there is a major hurdle I need to get past:

Problem (Andrew Paul): Given an initial velocity, angle, and gravitational acceleration, it was implied in the previous paper that the launch angles for each bounce change. This is because the Law of Reflection does not apply to collisions that are not perfectly elastic $(e<1)$. Create a function that gives the launch angle of the $n$th bounce given the aforementioned parameters.

Solution (Andrew Paul): The implication can be found in the equation:

$$
v_{n i} \sin \theta_{n}=e^{n-1} v_{i} \sin \theta_{1}
$$

Notice we write $\theta_{n}$ on the LHS of the equation. Simple thought experimentation and Figure 3.1 both suggest that as $n$ increases, $\theta_{n}$ decreases.

The key of course is to split up the initial velocity vectors of the projectile at the start of each bounce into its components. We know that the vertical component is given by $e^{n-1} v_{i} \sin \theta_{1}$. If we ignore nonconservative forces, there are no forces acting in the horizontal direction. Which means that the horizontal component is always $v_{i} \cos \theta_{1}$ regardless of the bounce. By simple SOHCAHTOA, this implies:

$$
\tan \theta_{n}=\frac{e^{n-1} v_{i} \sin \theta_{1}}{v_{i} \cos \theta_{1}}=e^{n-1} \tan \theta_{1}
$$

Hence we conclude:

$$
\theta_{n}=\arctan \left(e^{n-1} \tan \theta_{1}\right)
$$

Since $0<e<1$, we confirm that as $n$ increases, $\theta_{n}$ decreases.
We are on the verge of finding the bounce equation...


Fig 3.1: After each bounce, it appears that the initial launch angle decreases.

