# The 4$^{\text{th}}$ AMO4

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[*]Some problems were adopted from other sources.

This test consists of 5 problems, arranged from easiest to most difficult. Let $n$ be the problem number. Then, $n$ is also the maximum number of points possible on that problem. Hence, the maximum possible score on this test is $\sum_{k=1}^{5} k = 15$. Partial, clumsy, or non-elegant solutions on the $(n > 1)^{\text{th}}$ problem will earn a positive integer number of points less than $n$, the exact quantity of which will be decided by the grader. A PCNE solution on the first problem will result in 0 points.

No calculator of any kind is allowed *unless specified within the problem itself*. Compass and straightedge are allowed. $\frac{15-3n}{2}$ points will be deducted if the solver resorts to using any resources that is not his or herself on the $n^{\text{th}}$ problem *after points have been earned if that particular solution was correct*. There is no time limit on this test. Solutions to each problem must be formal, rigorous, and LaTeXed to be scored officially.

Good luck and have fun!

**PROBLEMS**

**Problem 1:** Curious Child: "How can I figure out if a number is divisible by three?"

American Teacher: "Well, you can just add up all the digits and see if that sum is divisible by three!"

Curious Child: "Really? Wow! Why does that work?"

American Teacher: "Umm... uhhh...."

Teach the child.

**Problem 2:** A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of the fourth side?

**Problem 3:** Evaluate:

$$\sum_{n=20}^{170} \sqrt{n + \frac{n^6 - n^3}{\sqrt{n + \frac{n^6 - n^3}{\sqrt{n + \ldots}}}}}$$

**Problem 4:** What is $\cos 36°$?

**Problem 5:** Consider the set of powers of 2:

$$\{1, 2, 4, 8, \ldots\}$$

Prove that every positive integer is expressible as the alternating sum of the elements of a subset of this set written in ascending order. (For example, $1 - 2 + 8 = 7$ and $-2 + 8 - 16 + 64 = 54$).