The 3rd AMO4

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*Some problem styles were adopted from other sources.

This test consists of 5 problems, arranged from easiest to most difficult. Let n be the problem number. Then, n is also the maximum number of points possible on that problem. Hence, the maximum possible score on this test is $\sum_{k=1}^{5} k = 15$. Partial, clumsy, or non-elegant solutions on the $(n > 1)^{\text{th}}$ problem will earn a positive integer number of points less than n, the exact quantity of which will be decided by the grader. A PCNE solution on the first problem will result in 0 points.

No calculator of any kind is allowed *unless specified within the problem itself*. Compass and straightedge are allowed. $\frac{15-3n}{2}$ points will be deducted if the solver resorts to using any resources that is not his or herself on the n^{th} problem *after points have been earned if that particular solution was correct*. There is no time limit on this test. Solutions to each problem must be formal, rigorous, and LaTeXed to be scored officially.

Good luck and have fun!

PROBLEMS

Problem 1: Is $5^{21}+2$ a perfect square? How about $5^{21}+1$?

Problem 2: A grid is said to be *pulchritudinous* if it is 4×4 and each 1×1 square within it is colored either blue or pink such that they are each adjacent to two of the same color. Must every pulchritudinous grid contain a 2×2 square of uniform (strictly blue or strictly pink) color?

Problem 3: Compute the sum of the factors of $7^{12}+324$.

Problem 4: Consider the equation:

$$(4x^2-92x+530)(y^4-70y^2+3242)=2017$$

Where $x, y \in \mathbb{R}^+$. Find the solution (x, y) and prove that it is the only existing solution.

Problem 5: Let $\triangle ABC$ have incenter I and suppose that $\overrightarrow{AI}$ hits the circumcircle of $\triangle ABC$ at L. Prove that the circumcenter of $\triangle IBC$ is L.