The 2nd AMO4

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*All problems are original

This test consists of 5 problems, arranged from easiest to most difficult. Let n be the problem number. Then, n is also the maximum number of points possible on that problem. Hence, the maximum possible score on this test is $\sum_{k=1}^{5} k = 15$. Partial, clumsy, or non-elegant solutions on the $(n > 1)^{\text{th}}$ problem will earn a positive integer number of points less than n, the exact quantity of which will be decided by the grader. A PCNE solution on the first problem will result in 0 points.

No calculator of any kind is allowed *unless specified within the problem itself*. Compass and straightedge are allowed. $\frac{15-3n}{2}$ points will be deducted if the solver resorts to using any resources that is not his or herself on the n^{th} problem *after points have been earned if that particular solution was correct*. There is no time limit on this test. Solutions to each problem must be formal, rigorous, and LaTeXed to be scored officially.

Good luck and have fun!

PROBLEMS

Problem 1: Starting somewhere on the Earth, you walk a kilometer north, a kilometer east, and then a kilometer south to end back right where you started. You look down at the ground. What color is the ground?

Problem 2: Why is ASS not a valid congruency postulate?

Problem 3: In chemistry, a molecule of methane (CH_4) is an example of a *tetrahedral molecule*. That is, the 4 hydrogen atoms are the vertices of a regular tetrahedron and the carbon atom lies at the center of the tetrahedron such that it is equidistant to every hydrogen atom. The *bond angle* of this molecule is defined as the angle two "vertex atoms" make with the central atom (for instance, let two hydrogen atoms exist at H_1 and H_2, and a carbon atom at C, in methane; then $\angle H_1CH_2$ is a bond angle). Calculate the bond angle of a tetrahedral molecule.

Problem 4: What is the smallest unattainable distance between a pair of points selected from five points within a unit square?

Problem 5: Compute the sum:

$$\sum_{k=1}^{n} \frac{2k^2 - 2nk - 3k + n + 1}{2n^3 + 3n^2 + n}$$