

# The 1<sup>st</sup> AMO4

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\*Compiled from various competitions as well as original problems

This test consists of 5 problems, arranged from easiest to most difficult. Let  $n$  be the problem number. Then,  $n$  is also the maximum number of points possible on that problem. Hence, the maximum possible score on this test is  $\sum_{k=1}^5 k = 15$ . Partial, clumsy, or non-elegant solutions on the  $(n > 1)^{\text{th}}$  problem will earn a positive integer number of points less than  $n$ , the exact quantity of which will be decided by the grader. A PCNE solution on the first problem will result in 0 points.

No calculator of any kind is allowed *unless specified within the problem itself*. Compass and straightedge are allowed.  $\frac{15-3n}{2}$  points will be deducted if the solver resorts to using any resources that is not his or herself on the  $n^{\text{th}}$  problem *after points have been earned if that particular solution was correct*. There is no time limit on this test. Solutions to each problem must be formal, rigorous, and LaTeXed to be scored officially.

Good luck and have fun!

## PROBLEMS

**Problem 1:** Read the rules carefully and compute the minimum possible score on this test.

**Problem 2:** Find all solutions to the system of equations:

$$\begin{cases} x + y - z = 0 \\ zx - xy + yz = 27 \\ xyz = 54 \end{cases}$$

**Problem 3:** Let  $a, b, c \in \mathbb{N}_0$ . Find the number of triples  $(a, b, c)$  satisfying:

$$a + b + c = \sum_{i=1}^{21} k_i$$

Where  $k_i$  are the coefficients in the expansion of  $(2x + 3y)^{20}$ .

**Problem 4:** Consider 3 circles such that each circle intersects the others the maximum number of times (2 intersections between each pair of circles). Prove that the 3 common chords of the circles (the segments formed by connecting each pair of intersection points between each pair of circles) are concurrent.

**Problem 5:** Ravi wishes to cross a circular lake with diameter 1 km. He can row across the water at a rate of 5 km/hour, or he can walk along the shore (carrying his boat) at a rate of 7 km/hour. What is the minimum amount of time necessary to cross the lake?